Review for Final Exam

Review Questions

1. When using IBP, $\int udv = uv - \int vdu$ we look for u to be easily ...., and v to be easily....

2. Review trig identities (remember the game we played?)

3. When we decompose rational functions, what are the denominators? Briefly explain the Heaviside method and give an example.

4. How do the Trapezoid rule differ from the Simpson's?

5 How do we define improper integrals? What are some common tests to determine if an improper integral converges or diverges?

6 What is a separable differential equation? First order linear DE? Explain the methods of solving each type and give examples.

7 Why is Euler method called a patch of linear approximations?

7 What is the difference between a series and a sequence?

8 Distinguish the convergence of a series and of a sequence? How are these two concepts related?

9 List all the methods you know about how to determine a convergence of a sequence?

10 What is the L'Hopital rule good for?

11 $\lim_{n\to\infty} x^n =$ ... when $|x| < 1$ and $=$ ... when $|x| > 1$.

12 Is this true $\lim_{n\to\infty}(1 - \frac{x}{n})^n = 1^\infty = 1$? Recall other common limits.

13 Distinguish a geometric series and a p-series. When does each converge?

14 What is a telescope? What is a 'telescoping' series?

15 Why is the n-term test called a divergence (and not convergence) test?

16 What are the conditions to use the integral test?

17 Compare the limit comparision test for series and improper integral.

18 What is the relation between the ratio and root test with a geometric series? At what value the test is inconclusive?

19 What do you have to check if using the alternating series test? Is an absolutely convergent series convergent? Why?

20 Review the summary of test on page 591.

21 How to determine the radius of convergence for a power series?

22 Distinguish Taylor series/Maclaurin series/Taylor polynomial/Binomial Series? What operations on power series can be applied for Taylor series?

23 Compare the estimation of error in approximation using the Taylor formula/Alternating Series Estimation theorem.

24 Do you remember anything we learned before prelim 1? Fundamental theorem of calculus? Volume/Arclength/Surface areas? u-substitution?

Practice exercises

1 Solve differential equations

a $\frac{dy}{dx} = x^2(y^2+1)$, $y(1) = 3$

b Find the integrating factor for $x^3\frac{dy}{dx} - x^4y = e^x$

More exercises (5,6 f2010) (6,7 practice 2010), (5,f2009), (11,12 sp2009)

2 Sequence: Determine convergence

a $a_n = (\frac{n-1}{n})^n$

b $a_n = \frac{n^{n^n}}{n!}$

c $a_n = \frac{(n+1)\cos n\pi}{\sqrt{n}-2}$

More exercises (2, practice fall 2010) (6b, f2009) (5, sp 2009)

2 Series: Determine convergence

a $\Sigma_1^\infty \frac{\sin(n^3)}{n^2}$

b $\Sigma_1^\infty \frac{6(n!)^2}{(2n)!}$

c Explain convergence of $\Sigma_1^\infty(-1)^{n+1}\frac{n-10\sqrt{n}}{n^2}$ Does it converge absolutely?

More exercises (3,fall 2010), (3, practice 2010), (6a,c,d,e,f f2009), (6,sp2009)

3 Power series

a Find the interval of convergence of $\Sigma_1^\infty(-1)^{n+1}\frac{x^{2n}}{(2n-1)2n}$. Let $f(x)$ be the function defined by this series. Find $f'(1/\sqrt{3})$

More exercises (11(a), fall 2010)(5,sp 2011)

4 Taylor series/Taylor series/Taylor polynomial

a Find the 3rd Taylor polynomial of $f(x) = \frac{1}{4-x}$ around $x = 2$

b Find the first 3 nonzero terms of the Maclaurin series of $\sin(2x^{10})$

More exercises: (6,sp2010)(4,f2010) (4,5,8 pr2010) (7,8 f2009), (9,10 sp2009)

5 Application of integration to calculate volume/surface area/arclength

a Let $y = f(x)$ be a solution to the DE $y' = \sqrt{3-x^2}$ Find the area of the surface generated by rotating the graph $y = f(x)$, $0 < x < 1$ around the

y-axis. Why doesn't the result depend on the choice of the solution?

More excercises (1,sp 2010), (8,f2010), (9,10 pr2010), (2,3,4 f2009), (3,4 sp 2009), (8,sp2011)

6 Integration technique
a $\int x^3 \ln x dx$
b $\int \frac{x^4}{x^2-4} dx$
c $\int \frac{dx}{\sqrt{x}-\sqrt[4]{x}}$
d $\int \sin^2 x \cos^2 x dx$

More exercises (1, f2010),(9, f2010) (6, sp 2011) (1,11 practice 2010), (1, f2009), (1,2 sp2009)

7 True of False and Explain
a $\Sigma_{n=1}^{\infty} a_n$ converges then $a_n \to 0$
b If an alternating series converges, the series consisting of its positive terms also converges.
c If a series converges absolutely, the series consisting of its positive terms also converges.

More exercises (10, f2010)(12, practice 2010)(7, sp 2009)

8 Other topics: Error estimates (11b, fall 2010) (4,8 practice 2010), Trapezoidal rule (4,practice 2010)(4 fall 2010), etc.